Active Structure Control

1 Presentation

1.1 Objectives

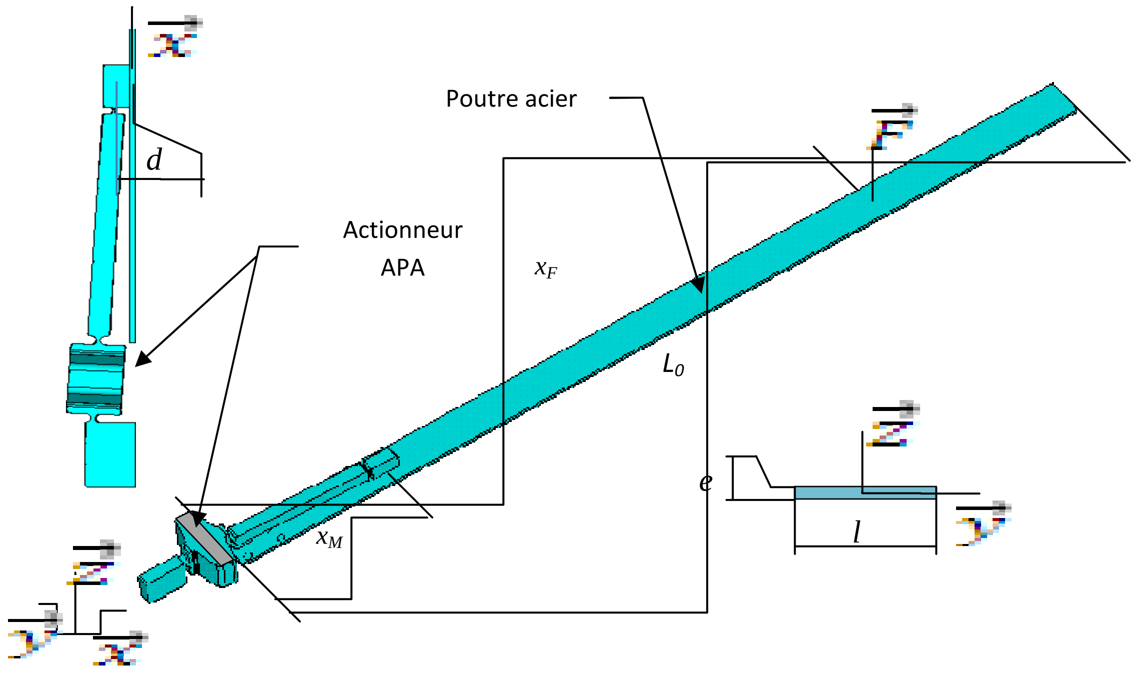
We are focusing on the vibratory behaviour of a beam-type structure.

The instrumentation of this as well as the addition of a control loop allows us to end up with a mechatronic system where the absorption can be actively controlled.

The creation of such a system is a multi-step process which corresponds approximately to the set of tasks to complete in your practical class:

1) Establishment of a mechanical model of the instrumented structure
   a) Establishment of an analytical model
   b) The method for reducing and simplifying the model
   c) Taking into account the piezoelectric actuator
2) Identification
3) Synthesis and set-up of an adaptive control
   a) Synthesis
   b) First start-up
   c) Optimization

1.2 Diagram and definition of the structure setting



Poutre -Beam

Actionneur APA -APA Actuator

Figure 1. Setting and structure diagram

Figure 1 shows the beam structure diagram, the piezoelectric actuator and its link arm. Note that l = 15 mm is the beam width, L_o = 330 mm its length and e = 2 mm its thickness.

The structure holds an APA type piezoelectric actuator used around the fitting as well as an accelerometer at the end.

2 Establishing a dynamic behaviour model

2.1 Establishing a model

We suggest here that you work starting from a simplified structure model: a free-fitted beam with the characteristics of Figure 1. Note that ρ is the specific gravity and E is the Young module of the material used (steel).

2.1.1 Writing the equations

Hypotheses :

- The straight section is not deformable
- The deformations remain minor

- Inertia of rotation is negligible compared to the inertia of rest
- The displacement of a point M from the beam is W(x,t)$\vec{z}$

Free from exterior influence, the dynamic equilibrium of a section of beam is given by the equation (1):

$$m\,W(x,t) + \frac{\partial^2}{\partial x^2}\left(E\,I\frac{\partial^2 W(x,t)}{\partial x^2}\right) = 0 \qquad (1)$$

Where $m = \rho\, l\, e$ is the linear density of the beam, $I = {l\,e^3}/{12}$ is the second moment of area of the straight section compared to the neutral axis.

The free vibration equation for the beam comes from the above equation by making a hypothesis about the contrapuntal motion of the pulse ω, $W(x,t) = w(x)\sin(\omega t)$:

$$-\,\omega^2\, m\, w(x) + \frac{d^2}{dx^2}\left(E\,I\frac{d^2 w(x)}{dx^2}\right) = 0 \qquad (2)$$

What are the conditions at the limits of the mechanical problem?

2.1.2 Inherent frequencies and modes of the beam

The solution to the equation (2) is in the form: $w(x) = a_1\sin(\beta x) + a_2\cos(\beta x) + a_3\sinh(\beta x) + a_4\cosh(\beta x)$

Where $\beta^4 = {\omega^2 m}/{E\,I}$.

Taking into account the conditions at the limits, an by looking for non-zero solutions that satisfy them, we arrive at the beams inherent frequencies.

$$f_{pi} = {1}/{2\pi}\sqrt{\left({\mu_i}/{L_0}\right)^4 {EI}/{m}} \qquad (3)$$

The first three values of μ_i are: μ_1 = 1,875 | μ_2 = 4,694 | μ_3 = 7,855

Calculate the first three inherent frequencies.

Experimentally, using an electromagnetic actuator, work out the resonance frequencies of the first modes, in the form f_{r1}, f_{r2} etc…

Compare the theoretical and experimental values and explain the discrepancies between them.

The fundamental natural modes of vibration can be expressed by arbitrarily choosing a_1 = 1, so:

$$\Phi_i(x) = \sin(\beta x) + a_2\cos(\beta x) + a_3\sinh(\beta x) + a_4\cosh(\beta x) \qquad (4)$$

Where the coefficients a_i (i = 2..4) are such that they satisfy the equation system (5):

(5)

The fundamental natural modes of vibration constitute an orthogonal vectoral base as a scalar product: $\int_0^{L_0} \phi_i(x)\phi_j(x)dx$. Also, the fundamental natural modes of vibration are defined to the nearest constant multiplier (previously, we arbitrarily chose $a_1 = 1$).

Demonstrate that we have the following (6) property (use the fact that the modes are the solutions to equation (2)):

$$\int_0^{L_0} \phi_i(x)\phi_j(x)dx = \begin{cases} \int_0^{L_0} \phi_i(x)\phi_i(x)dx, & i = j \\ 0, & i \neq j \end{cases} \qquad (6)$$

Express the equations of the three first modes so that $\Phi_i(L_0) = 1$.

Trace the form of the first three modes.

Using the finished element modeling software, it is possible to obtain more precisely the fundamental natural modes of vibration of a structure. Figure 2 Shows the form of the first three modes of the TP beam. The modes obtained show the same shapes as the simplified approach from the moment that the creation elements are able to be transmitted.

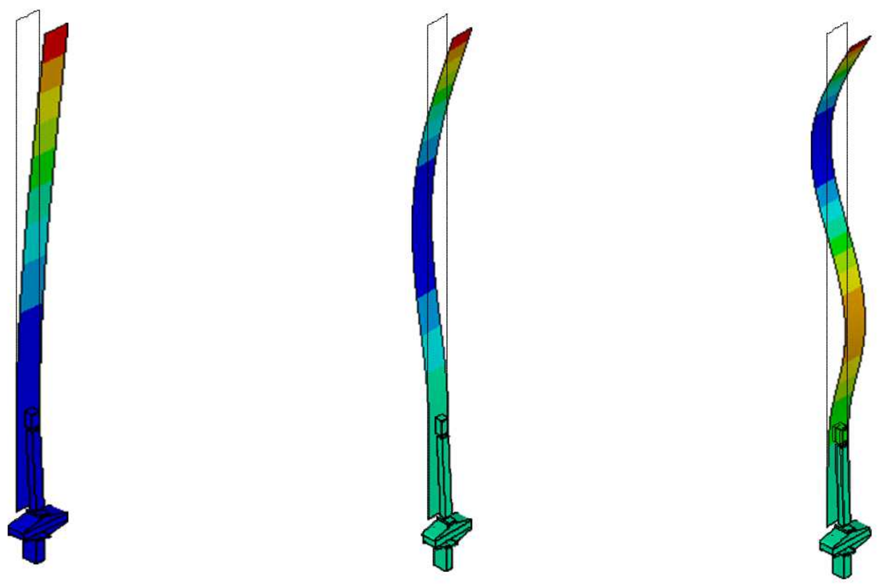


Figure 2. Forms of the first three fundamental natural modes of vibration of the structure

2.1.3 Taking into account the transversal force applied to point x_F

In the first approximation, the electromagnets effect can be modeled by a transversal force *F* in the direction $\vec{z}$ on the point of abscissa x_F (Figure 1). Taking this into account is done by modifying the second member of the dynamic equilibrium equation (1) in the following way:

$$m\,\ddot{W}(x,t) + E\,I\,\frac{\partial^4 W(x,t)}{\partial x^4} = F(t)\delta(x - x_F) \tag{7}$$

Where $\delta(x - x_F) = \begin{cases} 1, & x = x_F \\ 0, & x \neq x_F \end{cases}$, and we have $\int_0^{L_0} \delta(x - x_F)\Phi_i(x)\,dx = \Phi_i(x_F)$

2.2 Model reduction and simplification

2.2.1 Model reduction

The direct solution to equation (7) is difficult. A classic approach consists of looking for the solution by approximating the displacement field from several functions which satisfy the conditions with cinematic type limits (here the fitting). This is what we call the Ritz method.

By choosing the fundamental natural modes of vibration as approximation functions and by using the virtual work principal, we can bring back the solution to the previous equation from the 2nd order in time and the 4th order in space to a system of several equations uncoupled into 2nd order in time.

We thus state $W(x,t) = \sum_{i=1}^{n} q_i(t)\Phi_i(x)$, where $q_i(t)$ are the generalised co-ordinates and $\Phi_i(x)$ are the fundamental natural modes of vibration of the structure so that $\Phi_i(L_0) = 1$ ($i = 1..n$). *n* is the mode number chosen which particularly depends on the frequency band of interest. By replacing it in the equation (7), and by evaluating the virtual work for a displacement $\Phi_j(x)$, we obtain:

$$\int_0^{L_0} \left(\sum_{i=1}^{n} m\,\ddot{q}_i(t)\Phi_i(x)\Phi_j(x) + \sum_{i=1}^{n} EI\frac{q_i(t)(d^4\Phi_i(x))}{dx^4}\Phi_j(x) \right) dx = \int_0^{L_0} F\,\delta(x - x_F)\Phi_j(x)\,dx \tag{8}$$

By using the modes' orthogonal properties (equation (6)), show that we obtain *n* decoupled equations and give the M_i, K_i and β expression:

$$M_i\ddot{q}_i(t) + K_i q_i(t) = \beta_i F \tag{9}$$

We have thus successfully brought the solution to equation (7) back to a system of several uncoupled equations.

Trace the changes of coefficient β for the first three modes of the x_M ε [0, L_0] function:

Starting from the first mode frequency find experimentally the acceleration amplitude for x_M from 150mm to 200mm at intervals of 10mm. Did you find the changes predicted analytically?

Starting from the second mode frequency find experimentally the acceleration amplitude for x_M from 150mm to 200mm at intervals of 10mm. Did you find the changes predicted analytically?

What can you deduce from this about the position of the actuator?

The absorption does not appear in equation (7). In practice, from the moment the problem is under the form (9), we will add a modal absorption D_i of which the value is measured experimentally. The identification of this will take place later. We thus obtain n equations:

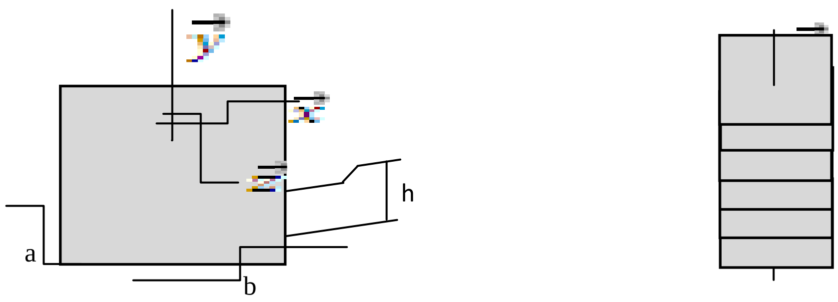
$$M_i \ddot{q}_i(t) + D_i\ \dot{q}_i(t) + K_i q_i(t) = \beta_i\ F \tag{10}$$

2.3 Taking into account the APA actuator and the transducer

2.3.1 APA actuator study

An APA actuator is composed of a stack of piezoelectric ceramics embedded in a metallic structure which enables amplification and constraint. Here we suggest focusing on the stack.

Each ceramic has a piezoelectric behaviour. The orientation of the piezoelectric properties as well as the electrodage of the ceramics are such that here we will only consider the 1D stack behaviour.



a) b)

Figure 3. Parametric transformation of a ceramic (a), ceramic stack (b)

The relations of the behaviour in direction $\vec{y}$ are:

$$\begin{cases} T_y = C_{Eyy}\ S_y - e_{yy}\ E_y \\ D_y = e_{yy}\ S_y + \ \varepsilon_{Sy}\ E_y \end{cases} \tag{11}$$

Where T_y is the traction-compression constraint, S_y is the deformation, E_y is the electric field and D_y is the electric displacement vector. ε_{Sy} is the deformation permittivity constant and e_{yy} is the piezoelectric coupling coefficient.

We provide the following relations:

- $E = {}^{V}/_{h}$
- $\int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} D_y \, dx \, dz = Q$ (electric equilibrium equation)

Integrate the behaviour relations (11) on a section of ceramic. By noting F_y as the normal effort (in direction $\vec{y}$) and by remembering that $S_y = {}^{\Delta y}/_{h}$, give the expression of the relations between F_y, Δy and V of one part, and Q, Δy and V of the other part.

The piezoelectric ceramics are assembled in parallel electrically and in series mechanically.

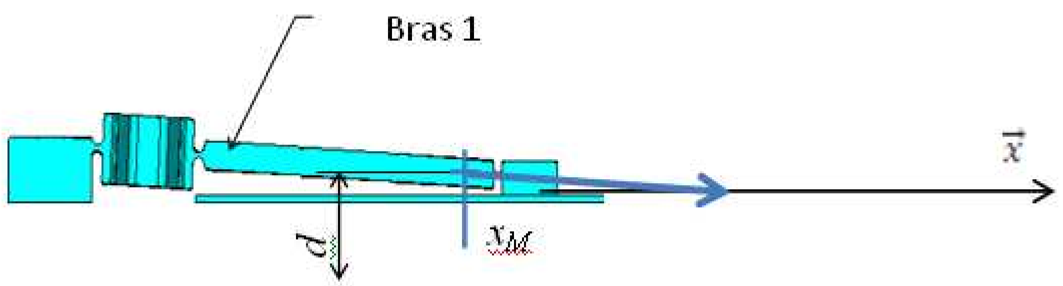
From previously obtained equations, determine the global behaviour equations for an assembly of n ceramics, and show that we then have:

$$\begin{cases} F_y = K_{éq}\,\Delta y - e_{yy}\,{}^{ab}/_{h}\,V \\ Q = e_{yy}\,{}^{ab}/_{h}\,\Delta y + C_{éq}\,V \end{cases} \qquad (12)$$

Give the expressions for $K_{éq}$ and $C_{éq}$.

The metallic structure of the APA allows the displacement amplification Δy of the ceramic assembly. We will write this amplification as A_p.

Taking into account the global architecture of the beam structure, and using the diagram in Figure 1, show that the effect of the actuator brings a traction-compression force and a bending moment Mp on the beam.



Express the displacement Δy as a function of the rotation of the section of the beam in x_M: ${}^{\partial W}/_{\partial x}(x_M, t)$

Give the expression of the relations between M_p, ${}^{\partial W}/_{\partial x}(x_M, t)$ and V of one part, and Q, ${}^{\partial W}/_{\partial x}(x_M, t)$ and V of another part.

Following this, we can make the hypothesis that the traction-compression and the bending behaviours are uncoupled. Thus, we do not have to consider the action of the actuator in traction-compression. The first inherent frequency of the traction-compression is much higher than the first inherent frequencies of the bending. The force along $\vec{x}$ vibrates the beam in a quasi-static manner, and its rigidity means that the effect is very weak.

2.3.2 Taking into account the bending moment applied to point x_M

Taking into account the bending moment localised in following the direction is done by modifying the second part of the dynamic equilibrium equation.

$$m\,\ddot{W} + E\,I\,\frac{\partial^4 W}{\partial x^4} = \frac{\partial M_p\,\delta(x - x_M)}{\partial x} \tag{13}$$

Where $\delta(x - x_M) = \begin{cases} 1, & x = x_M \\ 0, & x \neq x_M \end{cases}$

Apply the same method as i paragraph 2.2.1 and show that this time you obtain *n* time differential equations:

$$M_i\ddot{q}_i(t) + K_i q_i(t) = \gamma_i M_p \tag{14}$$

Show that $\gamma_i = \partial\Phi_i / \partial x(x_M)$ and trace the changes in γ_i as a function of x_M for the first three modes. What can you deduce?

To rigorously take into account the effect of the rigidity of the piezoelectric actuator, the fundamental natural modes of vibration must be recalculated for a mechanical structure which integrates the mechanical effect of the piezoelectric actuator and the arm (which enables an EF model). Thus, we can suppose here that the moment M_p is uniquely associated with piezoelectric coupling.

Finally, we have: $M_p = -\alpha\, d\, V$

Express the coefficient α as a function of A_p, a, b and e_{yy}.

2.3.3 Complete model

Here we are taking into account the structure, the APA and the electromagnetic actuator.

Write the mechanical equilibrium equation and the electric equilibrium equation, only considering the first mode.

Write the mechanical equilibrium equation and the electric equilibrium equation, only considering the first two modes.

3 Model identification

We are considering that the system behaviour is modeled by the equation system (15):

$$\begin{cases} M_1\ddot{q}_1 + D_1\,\dot{q}_1 + K_1\,q_1 - \alpha_1 V = \beta_1\,F \\ I = \alpha_1 \dot{q}_1 + C_{éq}\,\dot{V} \end{cases} \tag{15}$$

This model is valid as long as the beam is vibrated around its first resonance frequency.

- M_1 is the dynamic mass
- D_1 corresponds to mechanical losses
- K_1 corresponds to the rigidity of the system
- α_1 is the coefficient translating the electromagnetic coupling of the structure
- $C_{éq}$ is the piezoelectric actuator capacity
- q_1 is the displacement of the free end of the beam
- V is the tension on the piezoelectric actuator
- I is the current entering the piezoelectric actuator

This is to determine experimentally the parameters M_1, K_1, D_1, α_1 and C.

Using a capacitance meter, measure the electric capacity of the piezoelectric actuator. Compare this value with the value given in the APA specifications.

Using an electromagnet, vibrate the beam with the sinusoidal setting at its first resonance frequency f_{r1} , the piezoelectric actuator having been left as an open circuit. Observe synchronously the oscilloscope and the signal emitted by the accelerometer and the tension generated at the ends of the piezoelectric actuator.

From the second system equation (15), give the expression of the tension V as a function of q_1 when the actuator is part of an open circuit (I = 0)

Give the expression of the amplitude of the q_1 displacement as a function of the amplitude of the signal emitted by the accelerometer. Deduce from this the value of coefficient α_1.

The beam is now being vibrated by the the intermediary of the piezoelectric actuator, supplied with a sinusoidal tension of an amplitude of 20V and a variable frequency.

Read the amplitude of the beam displacement as a function of the vibration frequency for frequencies between $f_{r1}/2$ and 2 f_{r1}.

This reading corresponds to the frequential response the the function of the following transfer:

$$\alpha_1\, G_1(p) = \frac{\alpha_1}{M_1\, p^2 + D_1\, p + K_1} \tag{16}$$

We define the following characteristic dimensions associated with this transfer function:

- Inherent frequency (for which the phase difference is π/2): $f_{p1} = \frac{1}{2\pi}\sqrt{\frac{K_1}{M_1}}$
- Quality factor: $Q_1 = \frac{fr_1}{\Delta f} = \frac{K_1}{2\pi D_1 f_{p1}}$, where Δf is the band passing to -3dB

Determine f_{p1} and Q_1.

By observing that at the inherent frequency of the system, the transfer function module is equal to $\alpha_1/(2\pi\, D_1\, f_{p1})$, determine D_1. Deduce from this K_1 and M_1. What can we say about the mass compared to the mass of the beam?

4 Synthesis and set-up of an adaptive control

4.1 Theoretical principal

In the absence of active control (V=0), the transfer function between the exterior disturbing force and the displacement of the free end of the beam is given by the equation (18).

This presents a sharp resonance, which implies very great displacements from when the frequential specter of the disturbance coincides with the resonance frequency f_{r1}.

$$\Gamma_{BO}(p) = \beta_1\, G_1(p) \tag{17}$$

Show that $\Gamma_{BO}(p)$ can be put in the form of equation (18) below, and give the expressions of the parameters Q_{BO} and ω_{BO}.

$$\Gamma_{BO}(p) = \frac{\frac{\beta_1}{K_1}}{1 + \frac{p}{Q_{BO}\,\omega_{BO}} + \frac{p^2}{\omega_{BO}^2}} \tag{18}$$

The objective of the active vibration control is to modify this transfer function by attenuating as much as possible the resonance. The system control block diagram is represented in Figure 4 for which we define:

- $G_1(p)$: the function of the beam transfer (here we are only considering one mode)
- Sc : the sensitivity of the accelerometer (200mV/g)
- G_A : the gain of the power amplifier which supplies the actuator

- F_{ext} : the exterior disturbance (here the electromagnetic actuator)
- q_1 : the displacement of the free end of the beam
- $C(p)$: the corrector to synthesise in order to modify the vibratory behaviour of the beam

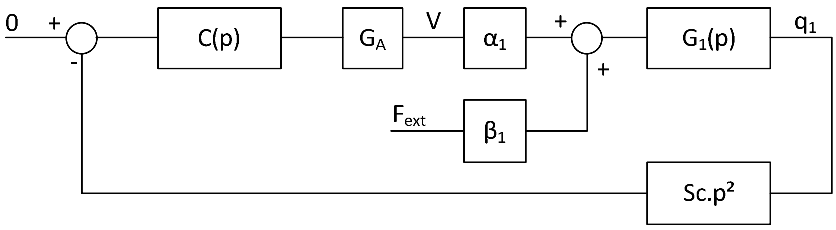


Figure 4. Principle of vibration control

Label each element of the block diagram and associate it with different elements of the experimental system.

Why is it labelled zero?

The corrector $C(p)$ is an integral proportional (IP) corrector, of which the transfer function is:

$$C(p) = P + {}^{I}\!/_{p} \tag{19}$$

Calculate the transfer function $\Gamma_{BF1}(p)$ between the exterior disturbing force and the displacement of the free end of the beam in a closed loop.

Show that this transfer function can be expressed in the following form:

$$\Gamma_{BF1}(p) = \frac{\frac{\beta_1}{K_1}}{1 + \frac{p}{Q_{BF}\omega_{BF}} + \frac{p^2}{\omega_{BF}^2}} \tag{20}$$

The term Q_{BF} is the quality factor of the closed loop system. A physical interpretation of this parameter is the approximate number of oscillations before returning to equilibrium. The pseudo-frequency of these oscillations will be proportional to ω_{BF}.

To have the fastest possible return to equilibrium, we want Q_{BF} small and ω_{BF} large.

Justify the choice of having Q_{BF} small and ω_{BF} large.

Propose a setting for corrector IP which enables the return to equilibrium in the two shortest possible pseudo-periods (the digital programming of the corrector forces us to have I and P positive or zero)

Using Matlab™ ou Scilab™, determine the gain and theoretical phase margins of the control diagram. Conclusions.

4.2 First set-up

We choose the following settings for the corrector *C(p)*:

- *P* = 0
- *I* = 2200
- No additional filter

Using the application HDPM45 (see Figure 5), set the corrector as indicated, then flip the switch to position « servo-on ». carry out a release. Is the system stable?

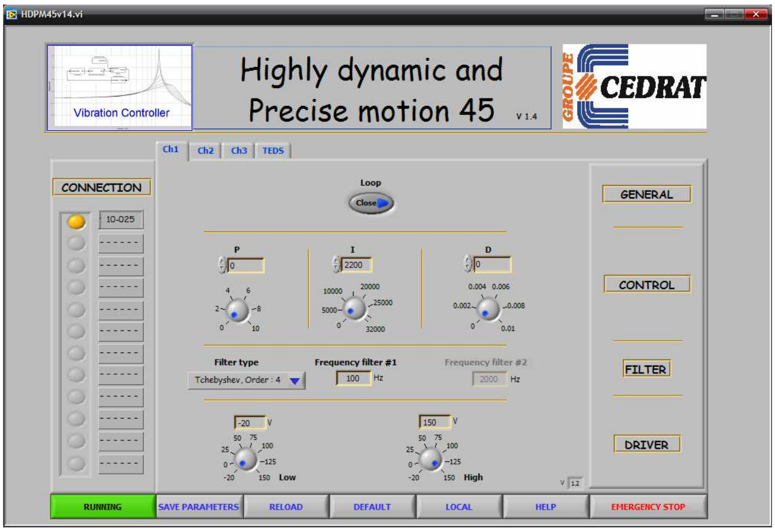


Figure 5. Application HDPM45

The observed instability cannot be explained if we only consider the first beam vibration mode. It is necessary to create a bimodal model of the beam to understand where this instability is coming from.

Using the same procedure as for the first mode, identify the parameters M_2, K_2, D_2 and α_2 associated with the second mode.

We then define the following transfer function, associated with mode 2:

$$\alpha_2\, G_2(p) = \frac{\alpha_2}{M_2\, p^2 + D_2\, p + K_2} \tag{21}$$

The block diagram of the system is represented by Figure 6.

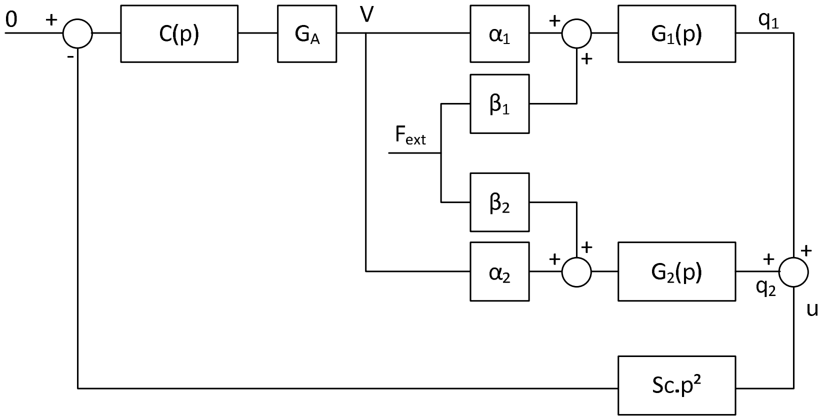


Figure 6. Principe of vibration control, 2-mode model

Calculate the transfer function $\Gamma_{BF}(p)$ between the exterior disturbing force and the displacement of the free end of the beam in a closed loop. Show that this transfer function can be expressed in the following way:

$$\Gamma_{BF_2}(p) = \frac{\frac{\beta_1}{K_1}}{1 + \frac{p}{Q_{BF1}\omega_{BF1}} + \frac{p^2}{\omega_{BF1}^2}} + \frac{\frac{\beta_2}{K_2}}{1 + \frac{p}{Q_{BF2}\omega_{BF2}} + \frac{p^2}{\omega_{BF2}^2}} \tag{22}$$

Trace the changes of Q_{1BF} and Q_{2BF} as a function of I. Show for which interval of I the control is stable. Conclusion.

4.3 Integral action filtering

The action of the integral corrector allows us to absorb the first mode, but tends to render the second unstable. One solution consists thus of filtering the corrector output in a way that will only affect the first mode.

We will choose an order-4 Tchebychev II type low-pass filter, of which the the cutting frequency is fixed at 100Hz. The transfer function of this filter is given by equation (23):

$$\mathrm{FT}_{100}(p) = \frac{0,03162\,p^4 + 99870\,p^2 + 3,943\;10^{10}}{p^4 + 1109\,p^3 + 618000\,p^2 + 2,021\;10^8\;p + 3,943\;10^{10}} \tag{23}$$

Trace the bode diagram of this filter, and explain why its use is well adapted to the system studied.

This filter is positioned after the integrator. The corrector *C(p)* is thus defined by:

$$C(p) = \left(P + \frac{I}{p}\right)\mathrm{FT}_{100}(p) \tag{24}$$

Trace the theoretical Bode diagram of the new transfer function $\Gamma_{BF2F}(p)$ and compare it with the Bode diagram of the open loop transfer function $\Gamma_{BO2}(p) = \beta_1\ G_1(p) + \beta_2\ G_2(p)$

Using Matlab™ or Scilab™, study the gain and phase margins. Can you reasonably increase the value of *I* in order to improve the performance?

We will lastly choose the following setting for the corrector *C(p)*:

- *P* = 0
- *I* = 2200
- Tchebychev filter at 100Hz

After having set the corrector according to the previous parameters, read the amplitude of the beam displacement as a function of the vibration frequency for the frequencies between $f_{r1}/2$ and $2f_{r2}$. Conclusion?

Carry out a release. How many oscillations does it take for the system to be absorbed? Conclusion?

4.4 Bimodal absorption

Following the previous observations, what should be the characteristics of a permanent filter to absorb simultaneously modes 1 and 2?

We will choose the following settings for the corrector *C(p)*:

- *P* = 0
- *I* = 2200
- Tchebychev filter at 500Hz

After having set the corrector according to the previous parameters, read the amplitude of the beam displacement as a function of the vibration frequency for the frequencies between $f_{r1}/2$ and $2f_{r2}$. Carry out another release. Conclusion?

Give the expression of the displacement amplitude q_1 as a function of the amplitude of the signal emitted from the accelerometer. Deduce from this the value of coefficient α_1.